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# Experimental and theoretical study of residual deformations and stresses at additive manufacturing by fusion

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## Abstract

Controlling the residual deformations and stresses at additive manufacturing is important to minimize the deviation of the part from the model and to avoid cracking. The tendencies and mechanisms of formation for residual stresses and deformations are studied at 3D printing with fused polymer. Horizontal beams are manufactured on thin vertical supports connecting them to a rigid substrate. The bending of the beam after detachment from the substrate is measured. The curvature radius appears to be proportional to the height of the beam and independent of the layer height. The thermoelastic theory applied to the multilayer beam confirm the experimental tendencies. The obtained results are common for other technologies adding melt layer-by-layer.

Keywords: selective laser melting; laser cladding; 3D printing; multilayer beam; bending

## 1. Introduction

Additive technologies of selective laser melting, multilayer laser cladding, and 3D printing by fused polymer are well-known examples of processes where a part is manufactured layer-by-layer from melt. When the solidified added material cools down to the ambient temperature, it shrinks forming tensile residual stresses. Experiments [1] confirm that the top of the part is under tension. Unloading at detachment of the part from the substrate causes additional deformation forming a tension at the bottom of the part. Finally, tensile residual stresses are at the top and at the bottom. They are balanced by compressive residual stresses in the middle of the part. Such three-zone stress distributions were measured in metal parts [1]. Brittle materials often crack because of the residual stresses. Studying the crack network gives useful information about the

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thermal shrinkage and suggests an estimate of residual stress distribution, which would form if the material were not cracked [2].

The formation of residual stresses is a disadvantage of the considered additive technologies because they deform the part relative the model and may crack brittle materials. To control the residual stresses, mathematical models were proposed for metals [3] and ceramics [4]. The models can give some useful recommendations to technologists. For example, increasing the ambient temperature inside the machine reduces the residual stresses. However, the existing models require a great number of parameters and are difficult to use. Plastic deformation of metals and cracking of ceramics complicate studying the mechanisms of residual stress formation. Thermomechanical properties of many polymer materials exclude both plastic deformation and cracking at additive manufacturing by fusion as follows from the estimation methodic [5]. Such materials can be described by the conventional thermoelastic model. This is why 3D printing by fused polymer is chosen for this study. In addition, the 3D printing technology is simple and flexible.

## 2. Experiment

Bridge-like samples were built from ABS resin. They consist of a rectangular-section beam mounted on multiple thin supports attached to a solid base as shown in Fig. 1a. The supports suppress bending of the beam during manufacturing but assure its free contraction/tension. The length $l$ and the width of the beams were 100 and 10 mm respectively. The height $H$ varied from 3 to 20 mm. The height of the layer $h$ was set to 200 or 350 mm. The supports connecting the beam and the base were delicately cut with a soldering iron. Residual stresses in the beam partially relaxed. This deforms the beam as shown in Fig. 1b. The ends of the beams deflected upward. The shape of the upper surface of the beams was measured by the contact method with multisensor coordinate measuring machine for stent measurement Werth SCOPE-CHECK 200.

The surface measured at $h = 200$ μm and $H = 3$ mm is shown in Fig. 2. The parabolic fit (see Fig. 2b) indicates that the upper surface is a circular cylinder and gives the curvature radius $R$. A strong linear correlation is found between beam height $H$ and curvature radius $R$ (see Fig. 3) while the variation of $R$ with layer height $h$ at constant $H$ is within the measurement uncertainty.

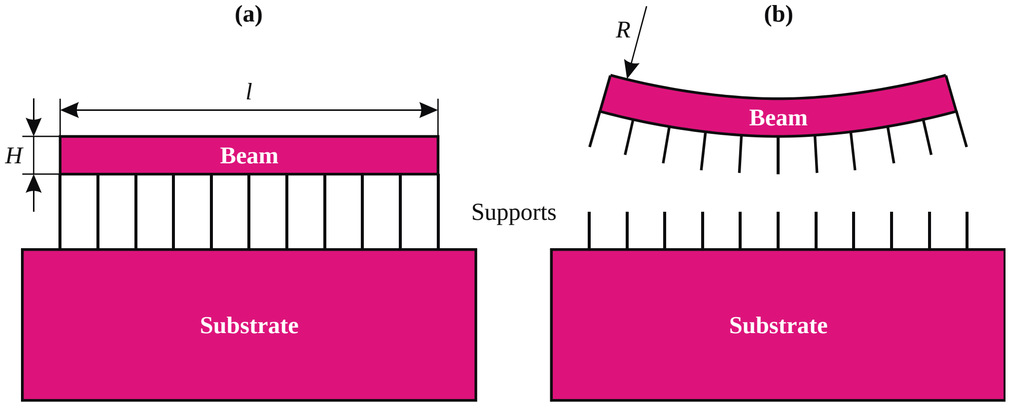

Fig. 1. Sample for studying the residual deformation: (a) as fabricated; (b) detached from the substrate

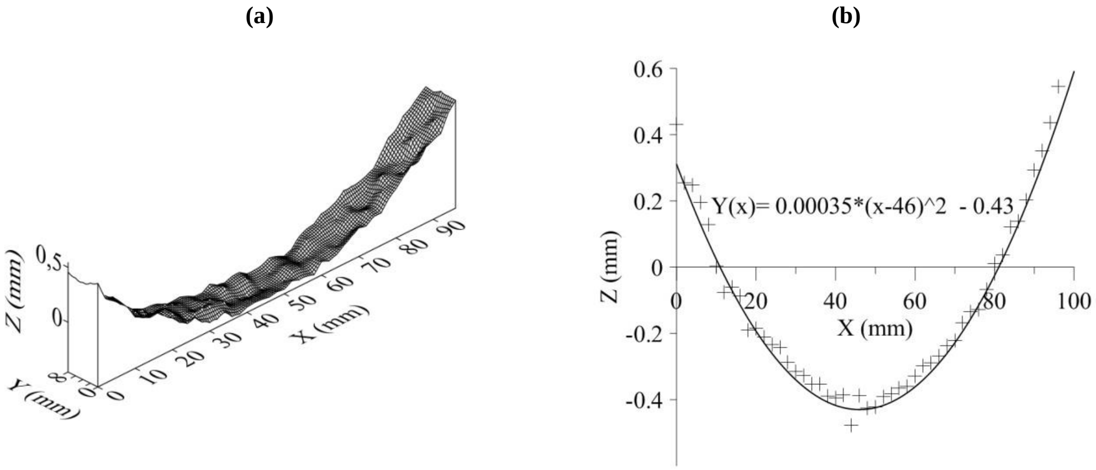

Fig. 2. Top surface of a beam: (a) 3D view; (b) average longitudinal profile (crosses) with the parabolic fit (line)

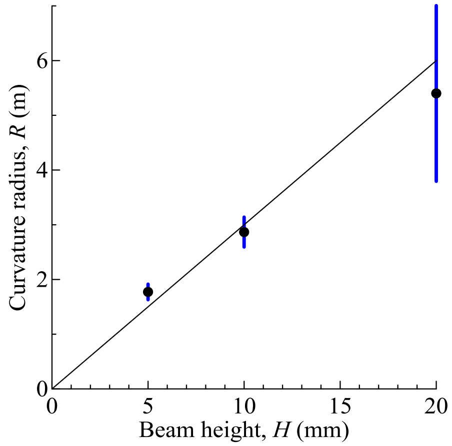

Fig. 3. Curvature radius versus the height of the beam. Points are the mean measured values. Vertical bars correspond to the mean square deviation. The line fits the experimental points

## 3. Theory and discussion

The theory developed for layered composites [6] can be adapted to describe bending of the obtained multilayer beams. Consider building of the beam by consecutive application of identic elastic layers. Each layer is applied on the top of the previous layer. Each layer shrinks just after its application because of cooling. The linear shrinkage of the layer would be

$$\varepsilon = \alpha(T_m - T_a)\,, \tag{1}$$

if the layer were not interact with their neighbours, where $\alpha$ is the linear thermal expansion coefficient, $T_m$ the melting (softening) point, and $T_a$ the ambient temperature. The force balance of $N$-layer beam reduces to

$$\sum_{i=1}^{N} \varepsilon_i = 0, \qquad (2)$$

where $\varepsilon_i$ is the residual deformation of $i$-th layer. Additional relations between the deformations follow from the consecutive character of their deposition [6]:

$$\varepsilon_{i+1} - \varepsilon_i = \frac{\varepsilon}{i}, \qquad (3)$$

where the layers are numbered from bottom (first deposited) to top (last deposited). The solution of Eqs. (2) and (3) is

$$\varepsilon_i = \varepsilon\left(1 - \sum_{j=i}^{N} \frac{1}{j}\right). \qquad (4)$$

The residual deformation of the first (bottom) layer $\varepsilon_1$ is always negative. The deformation of last (top) layer $\varepsilon_N$ is always negative. This indicates that the top of the $N$-layer beam is extended and the bottom is contracted. Thus, the beam tends to bend forming a concave top surface and a convex bottom one.

The final curvature radius formed after the beam detachment from the substrate can be found minimizing the elastic energy. The bending with curvature radius $R$ gives the additional deformations

$$\delta_i = \frac{h}{R}\left(\frac{N+1}{2} - i\right), \qquad (5)$$

where $h$ is the height of the layer. The elastic energy is

$$E = \frac{Fl}{2} \sum_{i=1}^{N} (\varepsilon_i + \delta_i)^2 + N \frac{Fl}{24} \frac{h^2}{R^2}, \qquad (6)$$

where $F$ is the rigidity of the layer defined as the product of the elastic modulus and the cross-section area of the layer. The first member of Eq. (6) is the sum of tension/contraction energies of the layers. The last member is the sum of bending energies. Finally, the minimum of Eq. (6) is attained at

$$\frac{H}{R} = 3 \frac{N-1}{N} \varepsilon. \qquad (7)$$

In the considered case of great number of layers $N$, theoretical Eq. (7) indicates that the curvature radius $R$ is essentially proportional to the beam height $H$. This corresponds to the experimental results shown in Fig. 3. In addition, the weak dependence of $R$ on $N$ at constant $H$ following from Eq. (7) indicates that $R$ is essentially independent of the layer height $h$. This is also confirmed by the experiments.

## 4. Conclusion

The beams manufactured layer-by-layer from melt accumulate residual stresses, which tend to bind them forming a concave top surface and a convex bottom one. The curvature radius $R$ measured after detachment of the beam from the substrate appears to be proportional to the beam height $H$ and independent of the layer height $h$. A theoretical approach is proposed to estimate the residual stresses and deformations based on the thermoelastic model of the material. The results are formulated as analytical formulas. They confirm the two revealed experimental tendencies.

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